תרגיל פונקציונאלי

1. $f:\mathbb{R}\to\mathbb{R}$ חסומה מלעיל, $f\left(x\cdot f\left(y\right)\right)+y\cdot f\left(x\right)=x\cdot f\left(y\right)+f\left(xy\right)$.

2. $f:\mathbb{R}\to\mathbb{R}$. $f\left(a^2\right)-f\left(b^2\right)\le\left(b+f\left(a\right)\right)\left(a-f\left(b\right)\right)$.

3. $f:\mathbb{R}_{>0}\to\mathbb{R}_{>0}$. $\left(z+1\right)f\left(x+y\right)=f\left(x\cdot f\left(z\right)+y\right)+f\left(y\cdot f\left(z\right)+x\right)$.

4. $f:\mathbb{R}_{>0}\to\mathbb{R}_{>0}$. $f\left(f\left(x\right)+x\cdot f\left(y\right)\right)=xy+f\left(x\right)$.

5. $f:\mathbb{Z}_{>0}\to\mathbb{Z}_{>0}$. $f\left(a\right)+b\mid\left(f\left(b\right)+a\right)^2$.

6. $f:\mathbb{R}\to\mathbb{R}$. $f\left(x^2+y+f\left(y\right)\right)=2y+\left(f\left(x\right)\right)^2$.

7. $f:\mathbb{R}\to\mathbb{R}$. $\left|y-f\left(f\left(x\right)\right)\right|\ge\left|\left(f\left(x\right)\right)^2+x\cdot f\left(y\right)\right|$.

8. $f:\mathbb{R}_{>0}\to\mathbb{R}_{>0}$. $f\left(1+x\cdot f\left(y\right)\right)=y\cdot f\left(x+y\right)$.

9. $f:\mathbb{R}\to\mathbb{R}$. $f\left(x+y\cdot f\left(x\right)\right)+f\left(x\cdot f\left(y\right)-y\right)=f\left(x\right)-f\left(y\right)+2xy$.

בהצלחה!